gain pattern. While the sidelobe structure of the computed patterns deviates somewhat from the ideal errorless case, the results are nevertheless indicative of excellent low-sidelobe performance over a large frequency band.

Conclusions

A coaxial waveguide amplitude commutator has been developed that can be employed in the design of a low-sidelobe scanning circular array antenna. The resultant antenna efficiency of such a design will be relatively high because the RF feed commutator network insertion loss is small and on the order of 0.5 dB. This waveguide commutator feed design is suitable for circular phased arrays that are employed as communication or radar antennas.

Acknowledgment

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Coupling Between Two Collinear Parallel-Plate Waveguides of Unequal Widths

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Abstract—**The problem of coupling between two collinear parallel-plate waveguides of unequal widths is investigated using the moment methods. The exciting mode of the waveguide is assumed as the incident field and an integral equation for the induced currents is expressed in terms of the reflected, the transmitted, and the evanescent currents on the walls of the waveguides. This integral equation is solved numerically and the results for the reflections and the transmission coefficients and the radiated field are obtained. The effect of varying the coupled waveguide width and the separation distance of the waveguides is investigated.**

I. Introduction

Both the Wiener–Hopf technique [1] and the moment method [2] were used by Elmoazzen to investigate the problem of coupling between two collinear parallel-plate waveguides of equal widths and separated by a certain distance. No attempt was made there to study the case of coupling between two waveguides of unequal widths. This work is devoted to the investigation of this problem. Since an exact solution is not known, a numerical method is used to solve the problem. It was shown in a previous paper [2] that an integral equation for the induced currents on the waveguides can be obtained in terms of the reflected, the transmitted, and the evanescent currents on the waveguides. The resulting integral equation was thus solved using the moment method [3] to find the desired currents. An application of the moment method to the problem reduces the integral equation to a set of linear simultaneous equations, the number of which depends on the number of propagating modes in the waveguides and the decay rate of the evanescent currents. Evanescent currents have significant values only near waveguide ends and need be considered only over a finite section of each waveguide end [4]. Once the reflection and the transmission coefficients as well as the evanescent currents are obtained, the radiation field can be obtained readily.

In the present work, a similar formulation is used to generate an integral equation for the currents on the waveguide walls. Pulse functions are utilized to describe evanescent currents and the integral equation is reduced to a set of simultaneous linear equations for the reflection and the transmission coefficients and the evanescent currents. The expressions for the elements of the coefficient matrix are obtained, which reduce to previous expressions [3] when waveguides have equal widths. The new expression for the radiation field is also given. Based on these expressions, a few numerical results are obtained and are presented in the last section.

II. Formulation of the Problem

A geometry of the problem is shown in Fig. 1. Assume a TE_{01} mode is propagating in the left waveguide, which has a width d. The coupled waveguide has a width $D=Fd$, where $F_{min}<F<\infty$. To insure mode propagation in the coupled waveguide, it is assumed that $F_{min}=\sin\theta_{01}=\lambda/2d$.

The incident, the reflected, and the transmitted currents on the walls of the exciting and the coupled waveguide are given by

$$J_z^i=\hat{z}\frac{1}{\eta}\sin\theta_{01}\exp(jkx\cos\theta_{01}) \tag{1}$$

$$J_z^r=\hat{z}\frac{R}{\eta}\sin\theta_{01}\exp(-jkx\cos\theta_{01}) \tag{2}$$

$$y=o,d, \qquad x>0$$

and

$$J_z^t=\hat{z}\frac{1}{\eta}\sum_{l=1}^{l_{max}}T_l\sin\theta_{0l}\exp(jkx\cos\theta_{0l}) \tag{3}$$

$$y=-\frac{1}{2}(D-d),\frac{1}{2}(D+d), \qquad x<-L$$

where $l=1,3,5,\cdots$, $\eta=120\pi$, $\theta_{0l}=\sin^{-1}(\lambda l/2D)$, and l_{max} is determined by the fact that $\sin\theta_{0l_{max}}$ must be smaller than unity. It is, therefore, given by $l_{max}=$integer of $(2Fd/\lambda)$. In these equations, R and T_l are the reflection coefficient of TE_{01} mode and the transmission coefficient of TE_{0l} mode, respectively. The evanescent modes also contribute to the induced currents on the waveguides, but are significant only near waveguide ends. Representing these evanescent currents by $J_z^e=\hat{z}J_z^e$, the boundary con-

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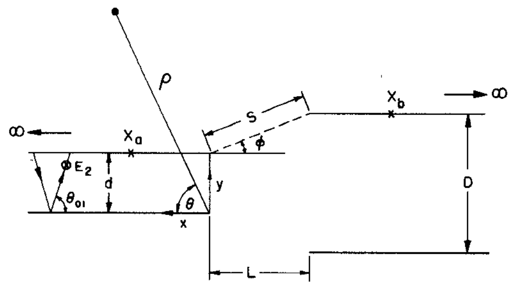

Fig. 1. Geometry of two coupled waveguides.

dition $E_z=0$ on waveguide walls gives the following integral equation for the induced currents [2]:

$$0=\frac{\eta}{4}\int_W \left[J_z^e(\bar{r}')+J_z^i(\bar{r}')+J_z^r(\bar{r}')+J_z^t(\bar{r}')\right] H_0^{(2)}(k|\bar{r}-\bar{r}'|)\, dkr' \tag{4}$$

where $\bar{r}$ and $\bar{r}'$ are the coordinates of the field and source points on the walls of the waveguide. The solution of this integral equation was discussed in [2] for two coupled waveguides of equal widths and is carried out here in a similar manner. The integrals involving the propagating modes are evaluated exactly and the evanescent-mode currents are represented by N pulse functions in the vicinity of waveguide open ends. Pulse functions are chosen as the basis functions only to simplify the evaluation of the integrals. In this manner, the integral equation (4) reduces to a set of simultaneous equations with $N+1+l_{\max}$ unknowns, $J_1^e, J_2^e,\cdots,J_N^e, R, T_1, T_2,\cdots,T_{l_{\max}}$. If F is less than $F_{\min}$, the number of unknowns reduces to $N+1$. In the matrix form, (4) may be written as

$$[l_{m,n}][f_n]=[g_m] \tag{5}$$

where

$$f_n=\begin{cases}\eta/4J_n^e, & n=1,2,\cdots,N\\ R, & n=N+1\\ T_{n-(N+1)}, & n=(N+1)+1,(N+1)+2,\cdots,(N+1)+l_{\max}\end{cases} \tag{6}$$

and the element of the excitation and the coefficient matrices are given by a) for test points on the exciting waveguides:

$$g_m=\exp(jkx_m\cos\theta_{01})\left\{\frac{\theta_{01}}{2\pi}-\frac{\sin\theta_{01}}{4}\cdot\left[\int_0^{kx_m} f_1(x,1)\,dkx-f_\infty(kd,1)\right]\right\} \tag{7a}$$

$$l_{m,N+1}=\exp(-jkx_m\cos\theta_{01})\left\{\frac{\theta_{01}}{2\pi}+\frac{\sin\theta_{01}}{4}\cdot\left[\int_{-kx_m}^{0} f_1(x,1)\,dkx+f_\infty(kd,1)\right]\right\} \tag{7b}$$

$$l_{m,N+1+l}=\frac{-\sin\theta_{0l}}{4}\exp(jkx_m\cos\theta_{0l})\cdot\left[\int_0^{kL+kx_m} f_2(x,l)\,dkx-f_\infty\left(\frac{kD-kd}{2},l\right)-f_\infty\left(\frac{kD+kd}{2},l\right)\right] \tag{7c}$$

b) for test points on the coupled waveguide:

$$g_m=\frac{-\sin\theta_{01}}{4}\exp(jkx_m\cos\theta_{01})\cdot\left[\int_0^{kx_m} f_2(x,1)\,dkx-f_\infty\left(\frac{kD-kd}{2},1\right)-f_\infty\left(\frac{kD+kd}{2},1\right)\right] \tag{8a}$$

$$l_{m,N+1}=\frac{\sin\theta_{01}}{4}\exp(-jkx_m\cos\theta_{01})\cdot\left[\int_{-kx_m}^{0} f_2(x,1)\,dkx+f_\infty\left(\frac{kD-kd}{2},1\right)+f_\infty\left(\frac{kD+kd}{2},1\right)\right] \tag{8b}$$

$$l_{m,N+1+l}=\exp(jkx_m\cos\theta_{0l})\cdot\left\{\frac{\theta_{0l}}{2\pi}-\frac{\sin\theta_{0l}}{4}\left[\int_0^{kL+kx_m} f_3(x,l)\,dkx-f_\infty(kD,l)\right]\right\} \tag{8c}$$

where

$$f_1(x,l)=\exp(-jkx\cos\theta_{0l})\left[H_0^{(2)}(|kx|)+H_0^{(2)}\left(k(x^2+d^2)^{1/2}\right)\right] \tag{9a}$$

$$f_2(x,l)=\exp(-jkx\cos\theta_{0l})\left[H_0^{(2)}\left(\left(kx^2+\left(\frac{kD-kd}{2}\right)^2\right)^{1/2}\right)+H_0^{(2)}\left(\left(kx^2+\left(\frac{kD+kd}{2}\right)^2\right)^{1/2}\right)\right] \tag{9b}$$

$$f_3(x,l)=\exp(-jkx\cos\theta_{0l})\left[H_0^{(2)}(|kx|)+H_0^{(2)}\left(k(x^2+D^2)^{1/2}\right)\right] \tag{9c}$$

$$f_\infty(YY,l)=\int_0^\infty \exp(-jkx\cos\theta_{0l})H_0^{(2)}\left(k(x^2+YY^2)^{1/2}\right)dkx. \tag{9d}$$

For $D=d$ one finds $f_3(x,l)=f_2(x,l)=f_1(x,l)=f_1(x,1)$ and (8a), (8b), and (7c) reduce to [2, (7a), (7b), and (7c)] for two waveguides of equal widths. The remaining matrix elements $l_{m,n}$ have

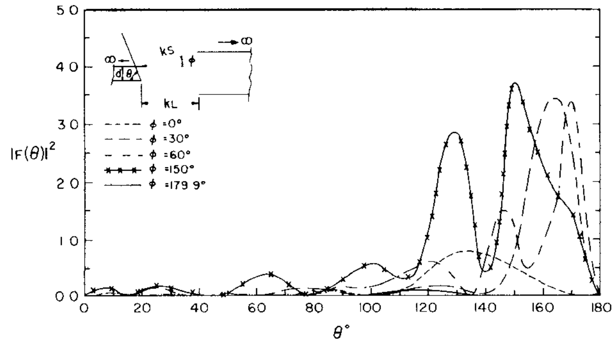

Fig 2 Radiation patterns for two collinear semi-infinite waveguides of unequal widths, $d=0.6\lambda$, $ks=10$.

the form

$$l_{m,n}=\begin{cases} \left[H_0^{(2)}(k|x_m-x_n|)+H_0^{(2)}\left(\left[(kx_m-kx_n)^2+(ks)^2\right]^{1/2}\right)\right]\Delta(kx_n), & \qquad (10a) \\ \qquad {x_m \atop x_n}<0, \quad m\neq n & \\ \left[1-j\frac{2}{\pi}\left(\ln\frac{\Delta(kx_n)}{4}+\gamma-1\right)+H_0^{(2)}(ks)\right]\Delta(kx_m), & \qquad (10b) \\ {x_m \atop x_n}<0, \quad m=n & \\ \left[H_0^{(2)}\left(\left[(kx_m-kx_n)^2+\left(\frac{kD-kd}{2}\right)^2\right]\right)^{1/2}+H_0^{(2)}\left(\left[(kx_m-kx_n)^2+\left(\frac{kD+kd}{2}\right)^2\right]^{1/2}\right)\Delta\right](kx_n), & \qquad (10c) \\ {x_m \atop -x_n}>0 \text{ or } {-x_m \atop x_n}>0 & \end{cases}$$

where $\gamma=0.5772\cdots$ and s is equal to d and D on the walls of the exciting and the coupled waveguides, respectively. The infinite integral in (9d) is evaluated using the Gauss–Laguerre quadrature formula [2].

The solution of (5) gives the evanesecent currents and the reflection and the transmission coefficients. To find the radiation field, the results of (5) are used in (4) with the left-hand side replaced by E_z. For the far radiation zone the result is

$$E_z(r)=\left(\frac{2}{\pi k\rho}\right)^{1/2}\exp(-jk\rho-\pi/4)F(\theta) \qquad (11)$$

where

$$\begin{aligned} F(\theta)=&\frac{j\sin\theta_{01}}{4}\left[1+\exp(jkd\sin\theta)\right] \\ &\cdot\left(\frac{1}{\cos\theta+\cos\theta_{01}}+\frac{R}{\cos\theta-\cos\theta_{01}}\right) \\ &-\frac{j}{4}\sum_{l=1}^{l_{\max}}T_l\frac{\sin\theta_{0l}}{\cos\theta+\cos\theta_{0l}}\exp\left[-jkL(\cos\theta+\cos\theta_{0l})\right. \\ &\left.+jk\left(\frac{D-d}{2}\right)\sin\theta\right]+\frac{\eta}{4}\sum_{n=1}^{N_1}J_n^e\exp(jkx_n\cos\theta) \\ &\cdot[1+\exp(jkd\sin\theta)]\Delta(kx_n) \\ &+\frac{\eta}{4}\sum_{n=N_1+1}^{N}J_n^e\exp(jkx_n\cos\theta)[1+\exp(jkD\sin\theta)] \\ &\cdot\exp\left[jk\left(\frac{D-d}{2}\right)\sin\theta\right]\Delta(kx_n) \qquad (12) \end{aligned}$$

where N_1 is the number of test points on the exciting waveguide. For $D=d$, these equations reduce to those given in [2] for two waveguides of equal widths. The interpretation of the results is similar to that given in [2] and is omitted here.

III. Results and Discussion

For a TE_{01} mode propagating in the exciting waveguide, some numerical results are obtained and are presented in this section. Fig. 2 shows the radiation patterns of two collinear semi-infinite waveguides with the exciting waveguide width $d=0.6\lambda$ and various widths and separations of the coupled waveguide. Since the radiation patterns of two coupled waveguides of equal widths, for different separations, was studied in [2] it is not considered here. Instead, in Fig. 2, the separation distance ks, between the edges of two waveguides is retained constant at $ks=10$, but their axial separation is changed as $kL=ks\cos\phi$. Thus for $\phi=0$, the problem reduces to the case of two coupled waveguides of equal widths and its radiation field is the same as that reported in [3], which was compared there with the result of the Wiener–Hopf technique [4]. With the above choice of kL, as ϕ increases from zero to $\pi/2$, the width of the coupled waveguide increases, while kL decreases. On the other hand, for ϕ in the range of $\pi/2$ and π, the coupled waveguide decreases in size and overlaps with the exciting waveguide. The problem then reduces to the radiation from the aperture generated between the two waveguides. In the limit when $\phi=\pi$, the coupled waveguide falls on the exciting one and the radiation terminates. This result is evident from Fig. 2, where for $\phi=179.9°$ the radiation field has reduced to negligibly small values.

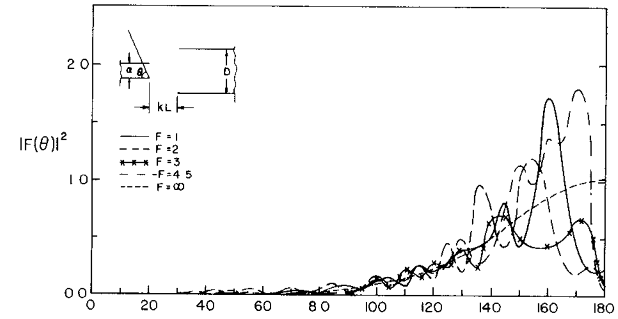

Fig. 3. Radiation patterns of two collinear semi-infinite waveguides of equal widths, $d=0.6\lambda$, $kL=50$, $F=D/d$.

TABLE I
REFLECTION AND TRANSMISSION COEFFICIENTS FOR $d=0.6\lambda$, $ks=10$

ϕ	Reflection Coefficient Mode 1		Transmission Coefficients					
			Mode 1		Mode 3		Mode 5	
	Amp.	Phase	Amp.	Phase	Amp.	Phase	Amp.	Phase
0°	0.285	-111°	0.5	-159°				
30°	0.142	-98°	0.28	-12°	0.17	-163°		
60°	0.163	-122°	0.18	10°	0.172	-179°	0.156	-8°
150°	0.193	-82°	0.188	34°	0.533	168°		
179.9°	0.108	-26°	1.05	36°				

TABLE II
REFLECTION AND TRANSMISSION COEFFICIENTS FOR $d=0.6\lambda$, $kL=50$

$F=\frac{D}{d}$	Reflection Coefficients Mode 1		Transmission Coefficients					
			Mode 2		Mode 3		Mode 5	
	Amp.	Phase	Amp.	Phase	Amp.	Phase	Amp.	Phase
1	0.199	-135°	0.261	141°				
2	0.182	-135°	0.146	-140°				
3	0.196	-134°	0.166	77°	0.082	164°		
4.5	0.213	-130°	0.161	29°	0.066	106°	0.066	-36°
∞	0.191	-134°						

To study the effect of coupled-waveguide width, the separation distance kL of two waveguides is retained constant, while kD the width of the coupled waveguide is modified. The computed radiation patterns are shown in Fig. 3. In this figure, $kL=50$, $d=0.6\lambda$, and the factor $F=D/d$ is changed from unity to infinity. The case of $F=1$ represents two coupled waveguides of equal widths, whereas $F=\infty$ represents an isolated open-ended waveguide. The results for different values of F oscillate with θ, the azimuthal angle, around the pattern of a single open-ended waveguide ($F=\infty$). The existance of the coupled waveguide produces multiple diffractions between the edges, which decreases in amplitude as the separation between the edges increases. Increasing F results in more propagating modes in the coupled waveguide and the number of simultaneous equations increases, which increases the computation time.

The reflection and transmission coefficients for the previous cases are shown in Tables I and II. The reflection coefficient fluctuates around that of a single waveguide and for $\phi=0°$ they are the same as those obtained in [2]. For $\phi=179.9°$, the transmission coefficient is almost unity, while the reflection coefficient is very small.

In conclusion, the moment method was used to study the coupling between two semi-infinite waveguides of unequal widths. An integral equation for the induced currents on the walls was obtained and was solved to give the reflection and the transmission coefficients and the evanescent currents. For certain selected data, radiation patterns were also computed.

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Slotted and Loose Braid Cables: Brief Conclusions of a Comparative Study

ANTÓNIO SIMÃO DE CARVALHO FERNANDES

***Abstract*—An analytical comparison is made of the electromagnetic characteristics of the coaxial mode of longitudinally slotted coaxial cables and loose braid coaxial cables in free space. Four aspects are considered: radial decay of the fields, percentage of power that travels outside the coaxial structure, characteristic impedance, and conductor loss.**

I. INTRODUCTION

Leaky cables are open structures that have been used to guide electromagnetic waves in continuous access communication systems [1], [2]. The loose braid outer conductor and the continuously longitudinally slotted outer conductor types are the two most used. These two types of leaky cables have been studied by several authors (e.g., Wait *et al.* [3]–[5], Fernandes [6]; Delogne *et al.* [7], Delogne [8], Hurd [9], Fernandes [10]). In a free-space situation the conclusions of these studies indicate that two fundamental, no cutoff frequency modes can propagate along these two structures: a coaxial mode (bifilar mode) whose energy is mainly (up to UHF) inside the coaxial structure with some leakage to the outside, and a monofilar mode whose energy is mainly outside, with some leakage to the inside of the coaxial structure.

For the loose braid cable, use was made of the surface transfer impedance concept (Z_{ST}) in order to characterize the braid [3]–[6], which was assumed thin with mesh dimensions small

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